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The New Integral Transform ''KUSHARE Transform''

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ABSTRACT

In this paper a new integral transform namely KUSHARE transform was applied to solve linear ordinary differential equations with constant coefficients.

Keywords: KUSHARE transform- Differential Equations.

I. INTRODUCTION

KUSHARE Transform is derived from the classical Fourier integral. Based on the mathematical simplicity of the KUSHARE transform and its fundamental properties. KUSHARE transform was introduced by Sachin Ramdas Kushare to facilitate the process of solving ordinary and partial differential equations in the time domain. Typically, Fourier, Laplace , Sumudu, Elzaki , Aboodh , Kamal , and Mohand transforms are the convenient mathematical tools for solving differential equations,Also KUSHARE transform and some of its fundamental properties are used to solve differential equations.

A new integral transform said to be KUSHARE change characterized for capacity of outstanding request we think about capacities in the set A characterized by

—

$$A = \{f(t)/\ \exists M, \tau_1, \tau_2 > 0, |f(t)| < Me^{\frac{|t|}{\tau_j}}, \text{if } t \in (-1)^j \times [0,\infty)\} \qquad (1)$$

For a given function in the set A, the constant M must be finite number, τ_1, τ_2 may be finite or infinite.KUSHARE transform denoted by the function f this transform has deeper Connection with the Mahgoub, Pourreza , Elzaki transform..The purpose of this study is to show the applicability of this interesting new transform and operator S (v) defined by the integral equations

$$K[f(t)]=S(v)=v\int_0^\infty f(t)e^{-tv^\alpha}dt, t\geq 0, \tau_1 \leq v \leq \tau_2 \qquad (2)$$

Where α is any non zero real numbers

The variable v in this vital change is utilized to figure the variable t the contention of the capacity v. This necessary change has further association with the Mahgoub, Pourreza, Elzaki changes. The reason for this examination is to show the pertinence of this intriguing new change and it productivity in tackling the direct differential conditions.

Notes:

(1) If α =1 then eq. (2) becomes

$$K[f(t)] = S(v) = v\int_0^\infty f(t)e^{-tv}dt, \quad t\geq 0, \quad \tau_1 \leq v \leq \tau_2$$

This integral transform is called "Mahgoub Transform".

(2) If α =2 then eq. (2) becomes

$$K[f(t)] = S(v) = v\int_0^\infty f(t)e^{-tv^2}dt, \quad t\geq 0, \quad \tau_1 \leq v \leq \tau_2$$

This integral transform is called "Pourreza Transform".

(3) If α = -1 then eq. (2) becomes

$$K[f(t)] = S(v) = v\int_0^\infty f(t)e^{-\frac{t}{v}}dt, \quad t\geq 0, \quad \tau_1 \leq v \leq \tau_2$$

This integral transform is called "Elzaki Transform".

The variable v in this transform is used to factor the variable t in the argument of the

its efficiency in solving the linear differential equations.

II. KUSHARE TRANSFORM OF THE SOME FUNCTIONS

i) $K(1) = \frac{1}{v^{\alpha-1}} = S(v)$

Inversion Formula : $K^{-1}\left(\frac{1}{v^{\alpha-1}}\right) = 1 = f(t)$

ii) $K(t^n) = \frac{\Gamma(n+1)}{v^{\alpha(n+1)-1}} = S(v)$

Inversion Formula : $K^{-1}\left(\frac{\Gamma(n+1)}{v^{\alpha(n+1)-1}}\right) = t^n = f(t)$

iii) $K(e^{at}) = \frac{v}{v^{\alpha}-a} = S(v)$

Inversion Formula : $K^{-1}\left(\frac{v}{v^{\alpha}-a}\right) = e^{at} = f(t)$

iv) $K(\sin at) = \frac{av}{(v^{2\alpha}+a^2)} = S(v)$

Inversion Formula : $K^{-1}\left(\frac{v}{(v^{2\alpha}+a^2)}\right) = \frac{\sin at}{a} = f(t)$

v) $K(\cos(at)) = \frac{v^{\alpha+1}}{(v^{2\alpha}+a^2)} = S(v)$

Inversion Formula : $K^{-1}\left(\frac{v^{\alpha+1}}{(v^{2\alpha}+a^2)}\right) = \cos(at) = f(t)$

KUSHARE Transform of derivatives :

(i) $K[f'(t)] = v^{\alpha}s(v) - vf(0)$

(ii) $K[f''(t)] = v^{2\alpha}s(v) - v^{\alpha+1}f(0) - vf'(0)$

(iii) $K[f^n(t) == v^{n\alpha}s(v) - v\sum_{k=0}^{n-1} v^{\alpha(n-k-1)}f^k(0)$

III. APPLICATION OF KUSHARE TRANSFORM OF ORDINARY DIFFERENTIAL EQUATIONS.

As stated in the introduction of this paper,

The initial conditions are

KUSHARE transform can be used as an effective tool. For analyzing the basic characteristics of a linear system governed by the differential equation in response to initial data. The following examples illustrate the use of KUSHARE transform in solving certain initial value problems described by ordinary differential equations.

Consider the first-order ordinary differential equation:

$$\frac{dx}{dt} + p(x) = f(t) \quad , \quad t > 0 \qquad (3)$$

With the initial condition

$x(0) = a$ (4)

Where p and a are constants and f(t)is an external input function so that its KUSHARE transform exists.

Applying KUSHARE transform of the Eq. (3) we have:

$$v^{\alpha}s(v) - vf(0) + p\,S(v) = \overline{f(v)}$$

$$S(v) = \frac{(\overline{f(v)} + av)}{(v^{\alpha} + p)}$$

The inverse KUSHARE transform leads to the solution.

Consider the second order linear ordinary differential equation has the general form:

$$\frac{d^2y}{dx^2} + 2p\frac{dx}{dt} + qy = f(x) \quad , \quad x>0 \qquad (5)$$

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$y(0) = a$, $y'(0) = b$ (6)
where p, q, a and b are constants. Application of KUSHARE transforms to this general initial value problem gives

$$v^{2\alpha}s(v) - v^{\alpha+1}f(0) - vf'(0) + 2p[v^{\alpha}s(v) - vf(0)] + qS(v) = \overline{f(v)}$$

The use of Eq.(6) leads to the Solution for S(v) as

$$S(v) = \frac{av^{\alpha+1} + vb + 2pva}{v^{2\alpha} + 2pv^{\alpha} + q}$$

The inverse KUSHARE transform gives the solution.

Example 3.1:
Consider the first order differential equation

$\frac{dy}{dx} + y = 0$, $y(0) = 1$ (7)

Applying the KUSHARE transform to both sides of this equation and using the differential property of KUSHARE transform , Eq.(7) can be written as:

$$v^{\alpha}s(v) - vf(0) + S(v) = 0$$

Where S(v) is the KUSHARE transform of the function y(x)
Applying the initial condition , we get
$(v^{\alpha} + 1)S(v) = v$ and $S(v) = \frac{v}{v^{\alpha}+1}$
Now applying the inverse KUSHARE transform , we get : $y(x) = e^{-x}$

Example 3.2:
Solve the differential equation

$\frac{dy}{dx} + 2y = x$, $y(0) = 1$ (8)

Applying KUSHARE transform to both sides of Eq.(8) and using the differential property of KUSHARE transform , Eq.(8) can be written as :

$$v^{\alpha}s(v) - vf(0) + 2S(v) = \frac{1}{v^{\alpha-1}}$$

$$(v^{\alpha} + 2)S(v) = v + \frac{1}{v^{\alpha-1}}$$

$$S(v) = \frac{v}{(v^{\alpha}+2)} + \frac{1}{(v^{\alpha}+2)v^{\alpha-1}}$$

The inverse KUSHARE transform of this equation gives the solution:
$y(x) = \frac{x}{2} + \frac{5}{4}e^{-2x} - \frac{1}{4}$

Example 3.3:
Let us consider the second–order differential equation
$y'' + y = 0$, $y(0) = y'(0) = 1$ (9)
Applying KUSHARE transform to both sides of Eq.(9) and using the differential property of KUSHARE transform , Eq.(9) can be written as

$$v^{2\alpha}s(v) - v^{\alpha+1}f(0) - vf'(0) + S(v) = 0$$

Applying the initial condition , we get

$$S(v) = \frac{v^{\alpha+1}}{(v^{2\alpha}+1)} + \frac{v}{(v^{2\alpha}+1)}$$

The inverse KUSHARE transform of this equation is simply obtained as
y(x)= sin(x) + cos(x)

Example 3.4:
Consider the following equation
$y'' - 3y' + 2y = 0$, $y(0) = 1$, $y'(0) = 4$ (10) Take KUSHARE transform of Eq.(10),
we find that: $v^{2\alpha}s(v) - v^{\alpha+1}f(0) - vf'(0) - 3[v^{\alpha}s(v) - vf(0)] + 2S(v) = 0$
Applying the initial condition , we

$$(v^{2\alpha} - 3v^{\alpha} + 2)S(v) - v^{\alpha+1} - 3v - 4v = 0$$

$$S(v) = \frac{v^{\alpha+1} + 3v + 4v}{v^{2\alpha} - 3v^{\alpha} + 2}$$

The inverse KUSHARE transform of this equation is simply obtained as
y(x)= $3e^{2x} - 2e^{x}$

Example 3.5:
Let the second order differential equation**:**
$y'' + 9y = \cos 2t$, $y(0) = 1$, $y(\frac{\pi}{2})$=-1 (11)

Since $y'(0)$ is not known, let $y'(0) = c$.
Take KUSHARE transform of this equation and using the conditions, we have

$$v^{2\alpha}s(v) - v^{\alpha+1}f(0) - vf'(0) + 9S(v) = \frac{v}{(v^{2\alpha}+4)}$$

$$(v^{2\alpha} + 9)S(v) = v^{\alpha+1} + c + \frac{v}{(v^{2\alpha}+4)}$$

$$S(v) = \frac{4}{5}\frac{v^{\alpha+1}}{(v^{2\alpha}+9)} + \frac{c}{5}\frac{3v}{(v^{2\alpha}+9)} + \frac{1}{5}\frac{v^{\alpha+1}}{(v^{2\alpha}+4)}$$

The inverse KUSHARE transform of this equation is simply obtained as
y(t)= $\frac{4}{5}cos3t + \frac{c}{5}sin3t + \frac{1}{5}cos2t$
To determine c note that $y(\frac{\pi}{2})$=-1 thin we find c= $\frac{12}{5}$
y(t)= $\frac{4}{5}cos3t + \frac{4}{5}sin3t + \frac{1}{5}cos2t$

Example 3.6:
Solve the differential equation**:**
$y'' - 3y' + 2y = 4e^{3t}$, $y(0) = -3$, $y'(0) = 5$ (12)
Taking KUSHARE transform both side of the differential Eq.(12) and using the given conditions we

$$v^{2\alpha}s(v) - v^{\alpha+1}f(0) - vf'(0) - 3[v^{\alpha}s(v) - vf(0)] + 2S(v) = \frac{4v}{v^{\alpha} - 3}$$

$$(v^{2\alpha} - 3v^{\alpha} + 2)S(v) = -3v^{\alpha+1} - 3v + \frac{4v}{v^{\alpha} - 3}$$

$$S(v) = \frac{-3v^{\alpha+1}}{v^{2\alpha} - 3v^{\alpha} + 2} - \frac{3v}{v^{2\alpha} - 3v^{\alpha} + 2} + \frac{4v}{(v^{\alpha} - 3)(v^{2\alpha} - 3v^{\alpha} + 2)}$$

Inverting to find the solution in the form.
$y(t) = 4e^{2t} + 2e^{3t} - 9e^{t}$

Example 3.7:
Find the solution of the following initial value problem:
$y'' + 4y = 12t$, $y(0) = 0$, $y'(0) = 7$ (13) Applying KUSHARE transform of this problem and using the given constants we

$$v^{2\alpha}s(v) - v^{\alpha+1}f(0) - vf'(0) + 4S(v) = \frac{1}{v^{2\alpha-1}}$$

$$(v^{2\alpha} + 9)S(v) = 7v + \frac{1}{v^{2\alpha-1}}$$

$$S(v) = \frac{7v}{v^{2\alpha} + 9} + \frac{1}{(v^{2\alpha} + 9)v^{2\alpha-1}}$$

Inverting to find the solution in the form
$y(t) = 3t + 2\sin 2t$

IV. CONCLUSION

In the present paper, a new integral transform namely KUSHARE transform was applied to solve linear ordinary differential equations with constant coefficients has been demonstrated.

REFERENCES

[1] Lokenath Debnath and D. Bhatta. Integral transform and their Application second Edition, Chapman & Hall /CRC (2006).
[2] G.K.watugala , simudu transform- anew integral transform to Solve differential equation and control engineering problems .Math .Engrg Induct .6 (1998), no 4,319-329.
[3] A.Kilicman and H.E.Gadain. An application of double Laplace transform and sumudu transform, Lobachevskii J. Math.30 (3) (2009), pp.214-223.
[4] J. Zhang, Asumudu based algorithm m for solving differential equations, Comp. Sci.Moldova 15(3) (2007), pp – 303-313.
[5] A. Kashuri and A. Fundo, “A New Integral Transform,” Advances in Theoretical and Applied Mathematics, Vol. 8, No. 1, 2013, pp. 27-43.
[6] Tarig M. Elzaki, (2011), The New Integral Transform “Elzaki Transform” Global Journal of Pure and Applied Mathematics, ISSN 0973-1768, Number 1, pp. 57-64.
[7] K. S. Aboodh, The New Integral Transform “Aboodh Transform” Global Journal of pure and Applied Mathematics, 9(1), 35-43(2013).
[8] Abdelilah Kamal. H. Sedeeg, The New Integral Transform “Kamal Transform”, Advances in Theoretical and Applied Mathematics, Vol.11, No.4, 2016, pp. 451-458.
[9] M. M. Mahgoub, The New Integral Transform “Mohand Transform”, Advances in Theoretical and Applied Mathematics, Vol.12, No.2, 2017, pp. 113-120.